



KATHOLIEKE UNIVERSITEIT  
**LEUVEN**

Colloque "Vibrations dans les bâtiments:  
sources, modes de propagation et techniques de réduction"  
CIDB - GIA - SFA  
22-23 March 2011, Paris, France



## Wave propagation in the soil

G. Degrande, M. Schevenels and S.A. Badsar

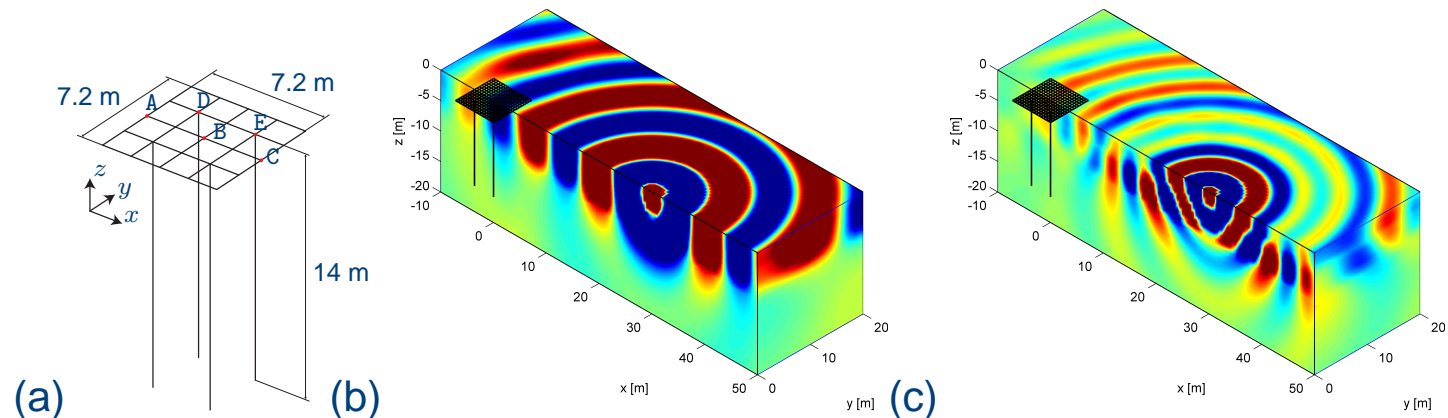
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[www.kuleuven.be/bwm](http://www.kuleuven.be/bwm)

Structural Mechanics, Department of Civil Engineering,  
K.U.Leuven

## Vibrations in the built environment

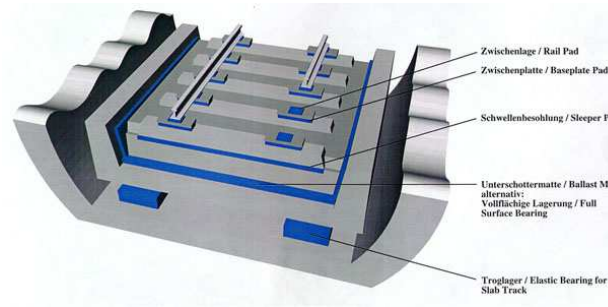
- **Response of a single family dwelling** (Retie, Belgium) due to the passage of a Volvo FE7 truck at a speed of 50 km/h on a traffic plateau, computed with a 3D coupled FE-BE model [Pyl et al., JEM, ASCE, 2003]
- **Response of the Bakerloo line tunnel** (Regent's Park, London) due to a moving train (single carriage) on a single wavelength unevenness, resulting in 40 Hz excitation, computed with a coupled periodic FE-BE model [Clouteau et al., JSV, 2005; Degrande et al., JSV, 2006].
- **Response of the Groene Hart tunnel** due to a non-moving harmonic load on the track at 40 Hz, computed with a coupled periodic FE-BE model [Gupta et al., SDEE, 2010].
- **Response of a piled foundation** (low vibration floors) for the Corelab 1B nanotechnology facility on the Arenberg III Campus (Heverlee, Belgium) [François et al., 2010].  
Real part of the vertical displacement in the soil and on the foundation due to a unit harmonic vertical point load at  $\{x = 30 \text{ m}, y = 0, z = 0\}^T$  at (b) 10 Hz and (c) 20 Hz.

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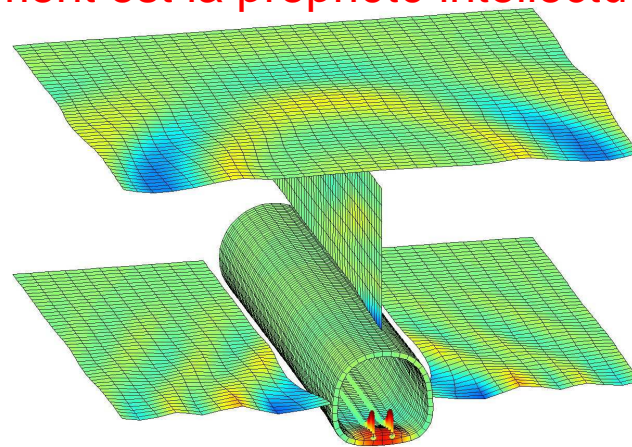
## Vibration mitigation at the source

### ■ Floating slab track:

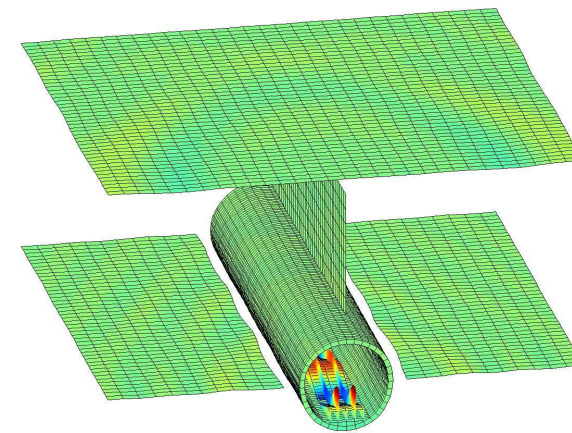


### ■ Harmonic response (60 Hz) of a track-tunnel-soil system (a) without isolation and (b) with a 10 Hz floating slab track.

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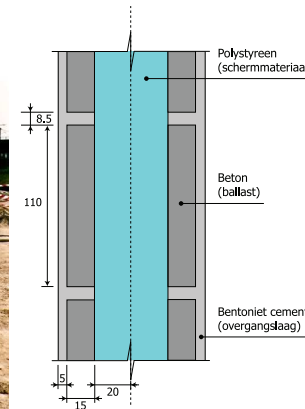
(a) Animation (avi) and zoom (avi).



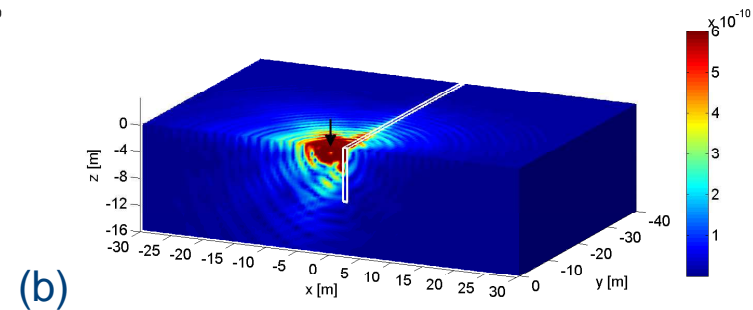
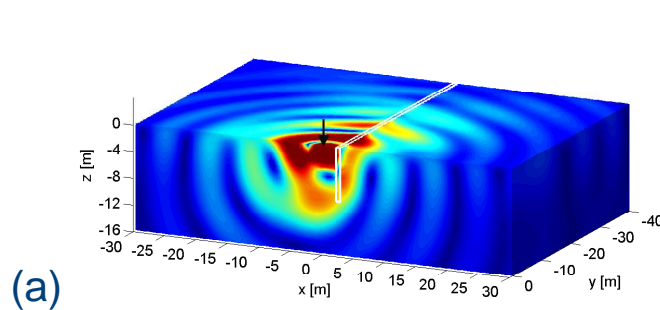
(b) Animation (avi) and zoom (avi).

## Vibration mitigation on the transmission path

- Vibration isolating (test) screen in Haren (1995):



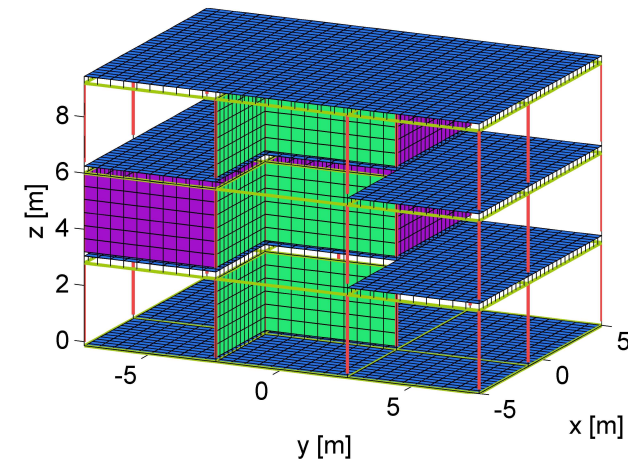
- Modulus of the real part of the displacement for a polystyrene screen at (a) 10 Hz (animation) and (b) 50 Hz (animation):



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## Vibration mitigation at the receiver

- Vibrations and re-radiated noise in an office building due to the passage of a train in a subway tunnel.



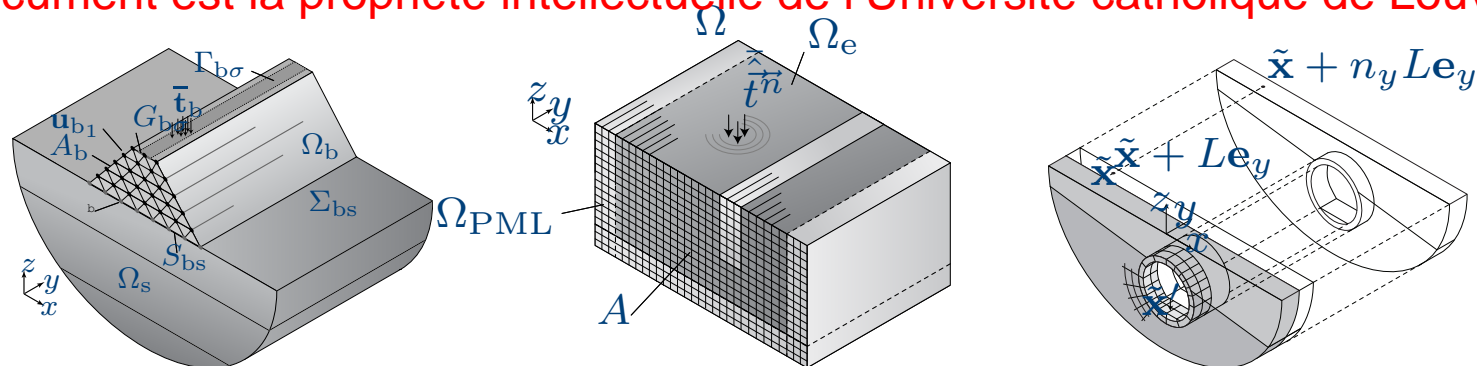
- ◆ Response of the unisolated structure.
- ◆ Response of the base isolated structure (10 Hz isolation).

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## Methodologies

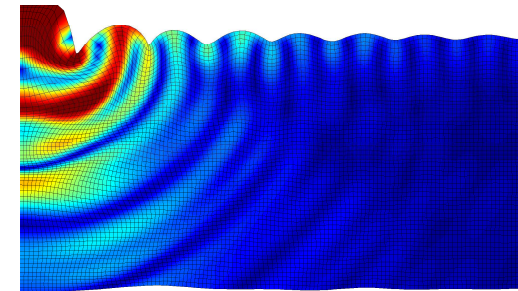
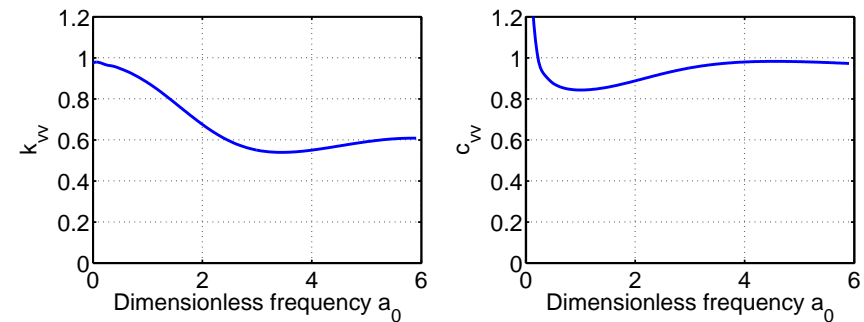
- 3D coupled FE-BE and FE-PML formulations for dynamic soil-structure interaction problems:
- Exploit the invariant or periodic geometry in the longitudinal direction of structures:
  - ◆ 2.5D FE-BE formulation (Fourier)  
[Aubry et al., WAVE, 1994; Andersen et al., JSV, 2006;...].
  - ◆ 2.5D FE-PML formulation (Fourier)  
[François et al., IJNME, submitted].
  - ◆ Periodic FE-BE formulation (Floquet)  
[Gupta et al., SDEE, 2007].
- Numerical tools have been developed and are used by several companies:
  - ◆ ElastoDynamics Toolbox (EDT) 3D and 2.5D Green's functions for layered media [Schevenels et al., CG, 2009].
  - ◆ TRAFFIC: vibrations in the built environment due to road traffic and rail traffic at grade and in tunnels.

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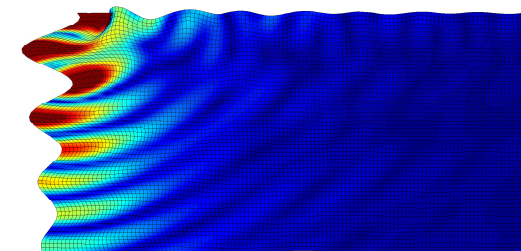
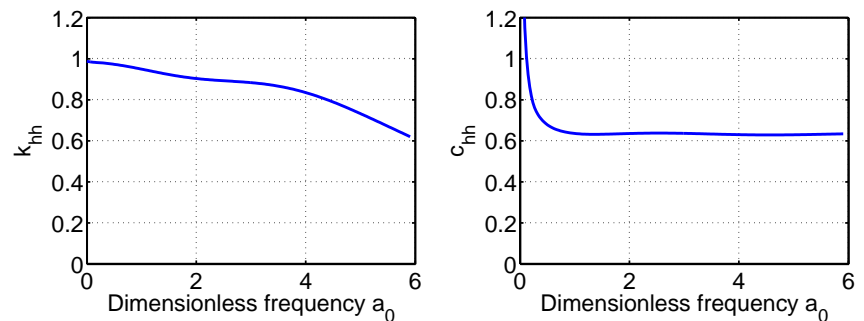


## Harmonic excitation of a rigid massless surface foundation

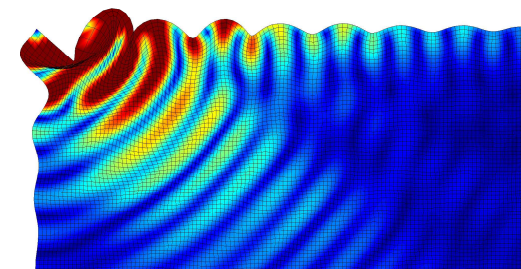
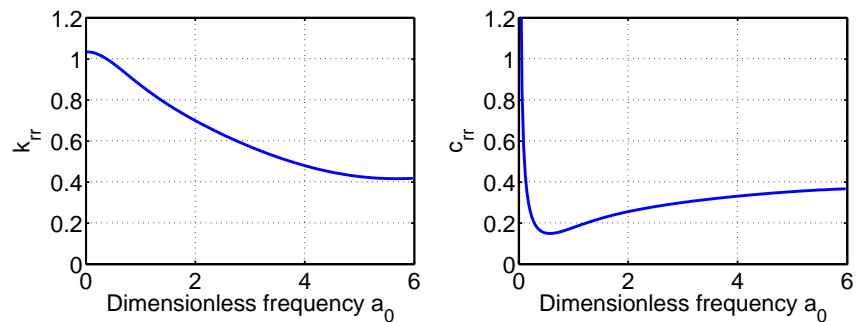
- Real (left) and imaginary (middle) part of the (a) vertical, (b) horizontal, and (c) rocking impedance of a rigid massless foundation and radiated wave field in the soil (right) at  $a_0 = 4$ .



(a)



(b)



(c)

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## Geometrical and material damping in a homogenous halfspace

- Plane harmonic wave:

$$\hat{u}(r, \omega) = Ar^{-n} \exp\left(-2\pi\beta\frac{r}{\lambda}\right) \exp\left(i\omega\left(t - \frac{r}{C}\right)\right) \quad (1)$$

- Geometrical damping:  $n = 0.5$  for Rayleigh waves and  $n = 2$  (at the surface) or  $n = 1$  (in the interior) for body waves.
- Material damping ratio (correspondence principle):

$$(\lambda + 2\mu)^* = (\lambda + 2\mu)(1 + 2i\beta_p) \quad (2)$$

$$(\mu)^* = \mu(1 + 2i\beta_s) \quad (3)$$

- Example ( $C = 150$  m/s,  $\beta = 0.025$ ,  $r_1 = 1$  m):

	4 m		16 m		64 m	
	Geom	Mat	Geom	Mat	Geom	Mat
10 Hz	0.500	0.969	0.250	0.855	0.125	0.517
50 Hz	0.500	0.855	0.250	0.456	0.125	0.037
100 Hz	0.500	0.730	0.250	0.208	0.125	0.001

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Problem outline

Elastic wave propagation

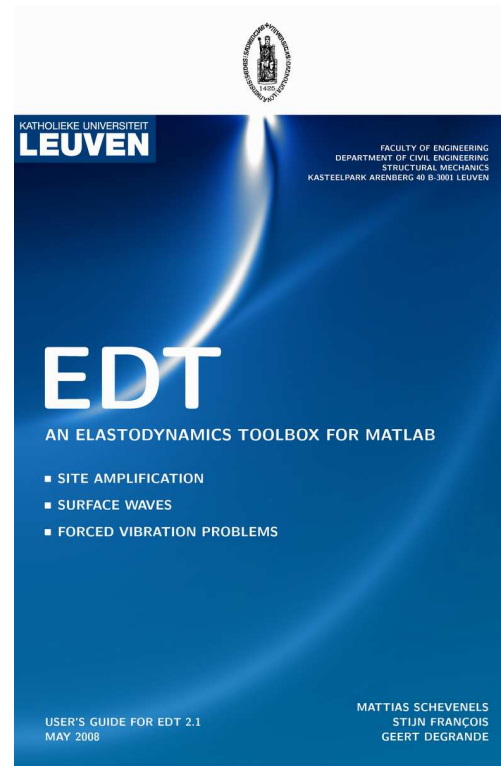
Dynamic soil characteristics

Case history

## EDT: An ElastoDynamics Toolbox for MATLAB

- Amplification problems, surface waves, forced vibration problems.
- Based on the direct stiffness method and thin layer method.
- Suitable for educational purposes, but also usable in a research environment.
- EDT and the user's manual can be obtained from:

<http://bwk.kuleuven.be/bwm/edt>



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## EDT: An ElastoDynamics Toolbox for MATLAB

- The direct stiffness method and the thin layer method can be used to solve a variety of problems governed by wave propagation in the soil:

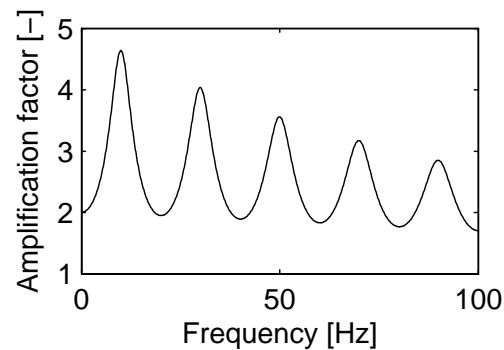
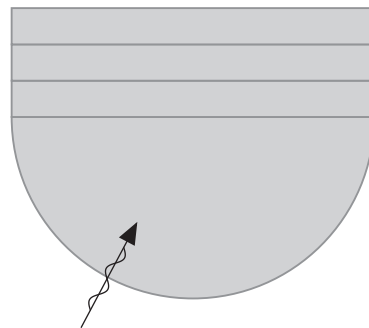
Problem outline

Elastic wave propagation

Dynamic soil characteristics

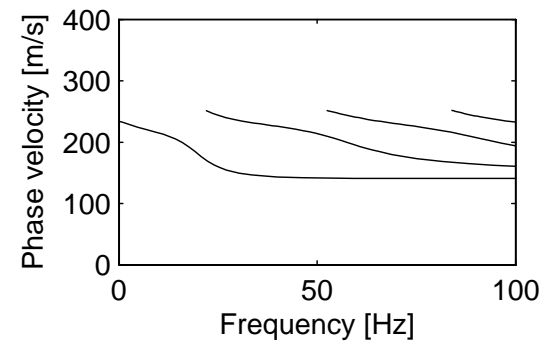
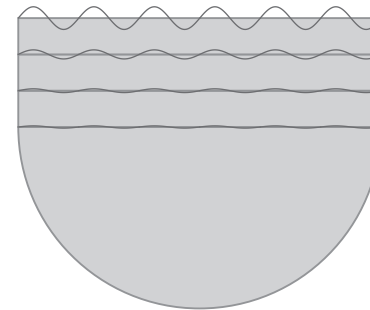
Case history

Site amplification



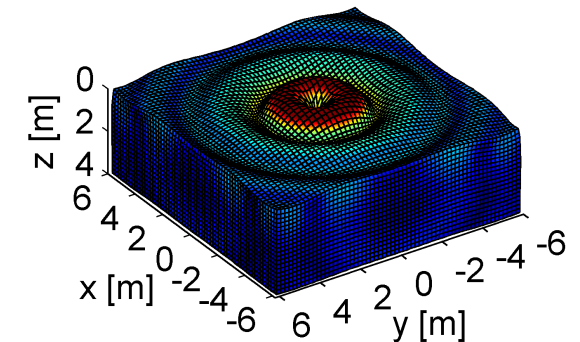
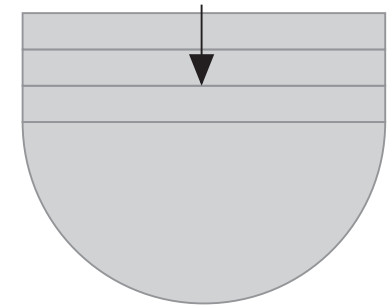
Amplification at the surface due to an incident P-wave.

Surface waves



Phase velocity of the Rayleigh waves.

Forced vibration problems



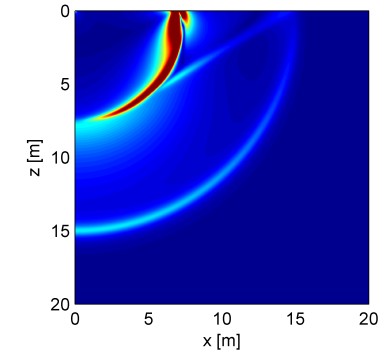
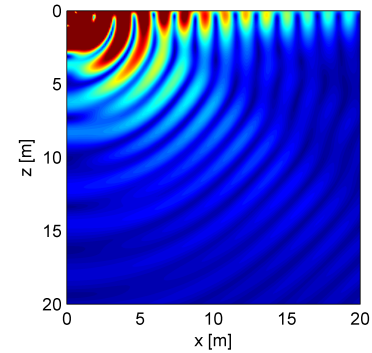
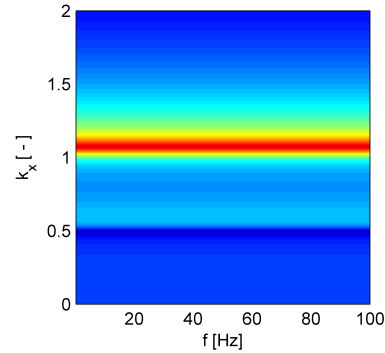
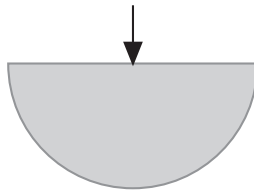
Response due to a harmonic point load at the surface.

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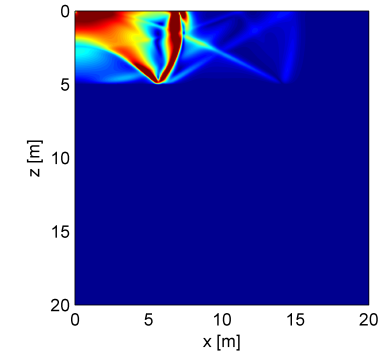
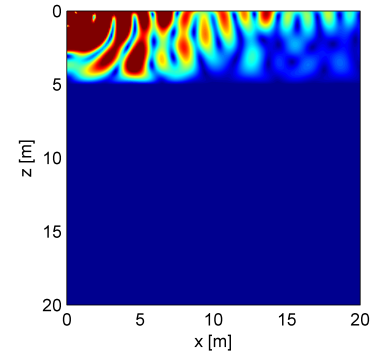
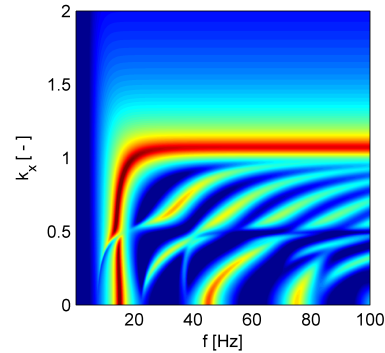
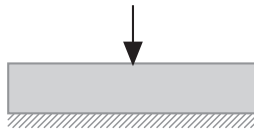
## Calculation of the Green's functions of soils

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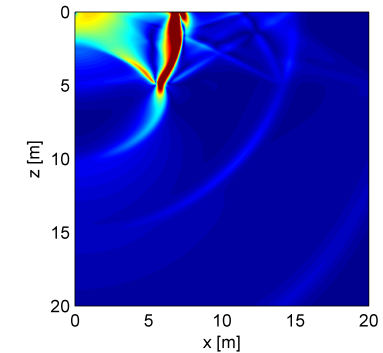
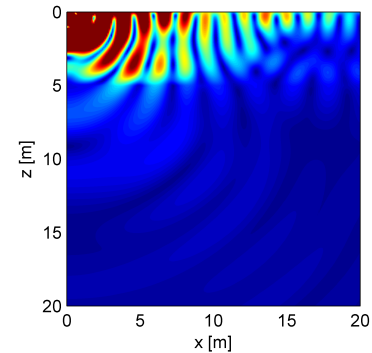
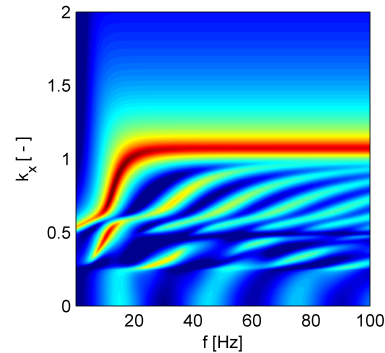
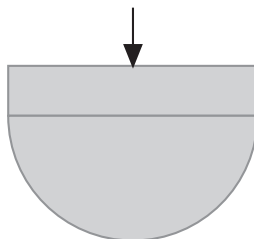
### ■ Homogeneous halfspace



### ■ Layer on bedrock



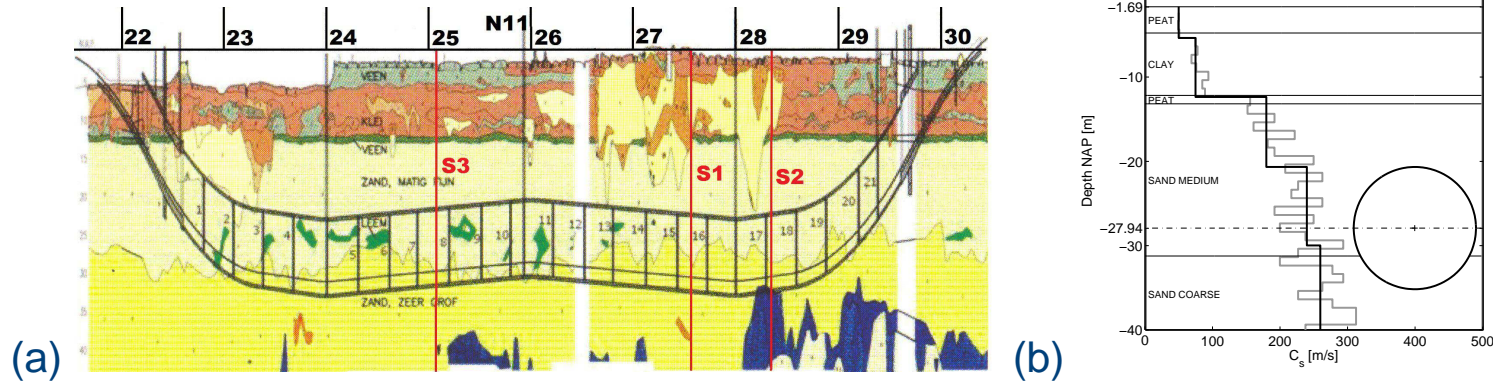
### ■ Layer on a halfspace



## Example 1: the Groene Hart tunnel

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- (a) Geotechnical profile and (b) shear wave velocity profile from SCPT at S3.



- The Groene Hart area is marshy and the soil is completely saturated.
- Top layers mainly consist of peat and clay, underlying half space is mainly sand.
- Tunnel depth at S3 is 26.25 m.
- P-wave velocity  $C_p$  from Biot's theory, density from classical soil mechanics test.
- 4 layers on top of a half space are considered for numerical calculations.

layer	thickness	$C_s$	$C_p$	$\rho$	$\nu$	$\beta$
	[m]	[m/s]	[m/s]	[kg/m <sup>3</sup> ]	[-]	[-]
1	3.7	50	1761.0	1107.1	0.4996	0.025
2	7.0	75	1719.3	1500.0	0.4990	0.025
3	8.3	180	1685.5	1970.6	0.4942	0.025
4	9.3	240	1715.1	1970.6	0.4900	0.025
5	$\infty$	260	1726.1	1970.6	0.4884	0.025

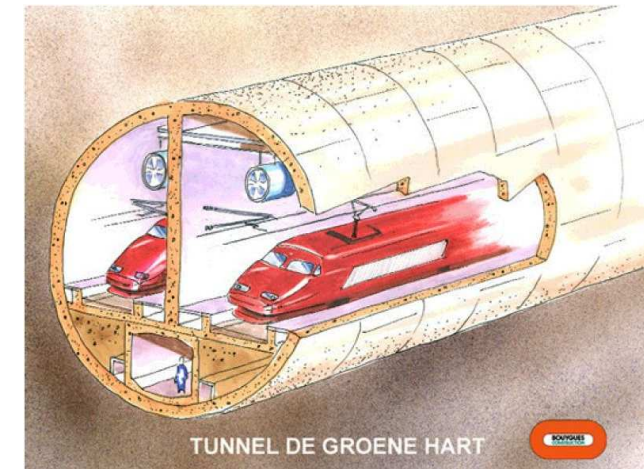
## Example 1: the Groene Hart tunnel

Problem outline

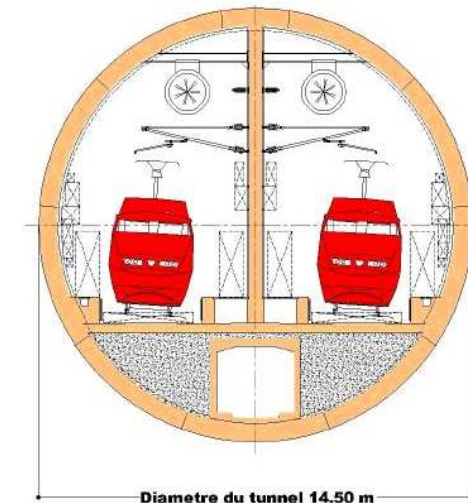
Elastic wave propagation

Dynamic soil characteristics

Case history



COUPE TRANSVERSALE DU TUNNEL



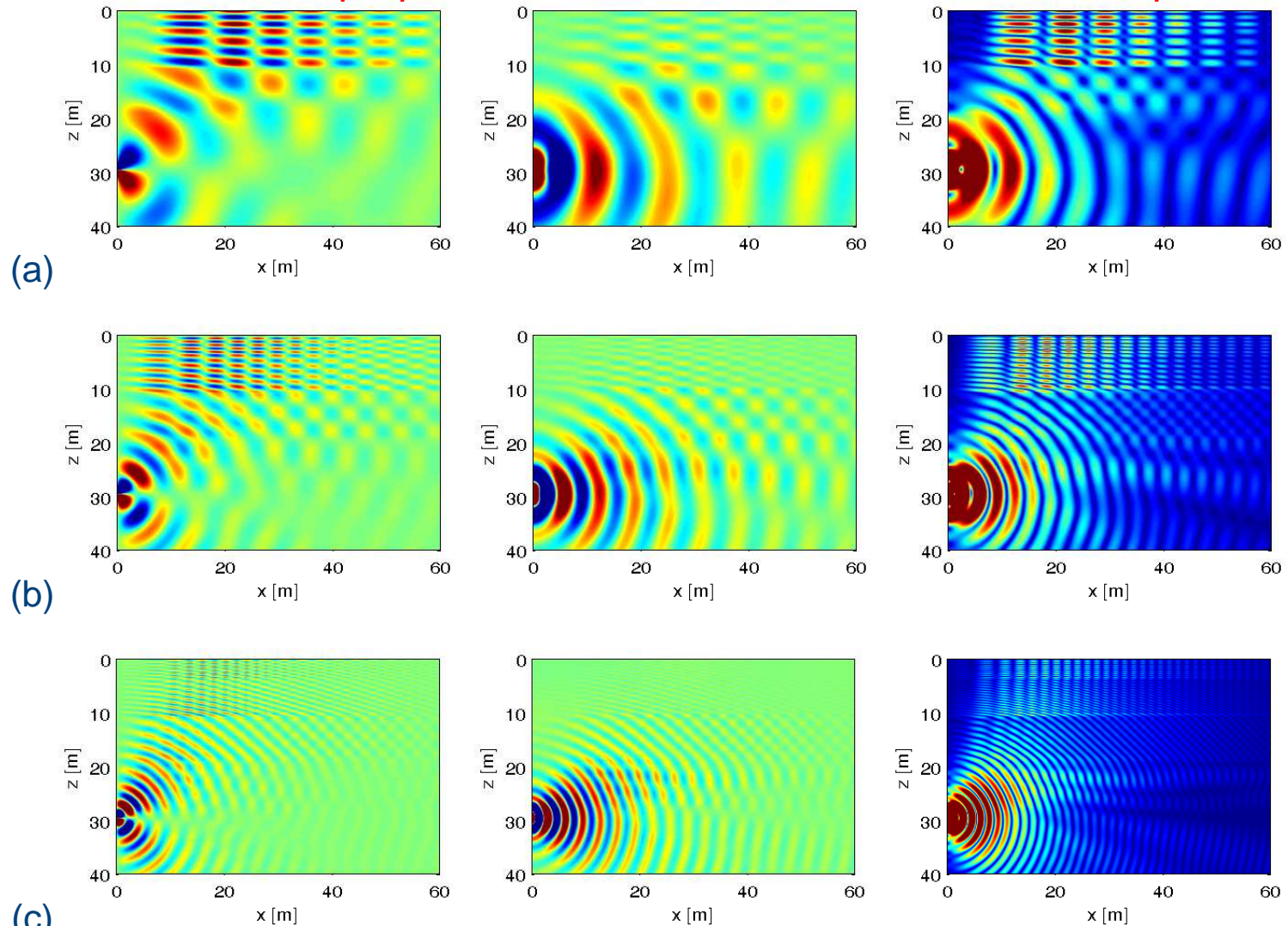
- Constructed using slurry shield tunneling method.
- Tunnel lining consists of prefabricated concrete sections.
- Technical gallery (duct).
- Sand/cement stabilization.
- Concrete floor and partition wall.

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## Example 1: the Groene Hart tunnel

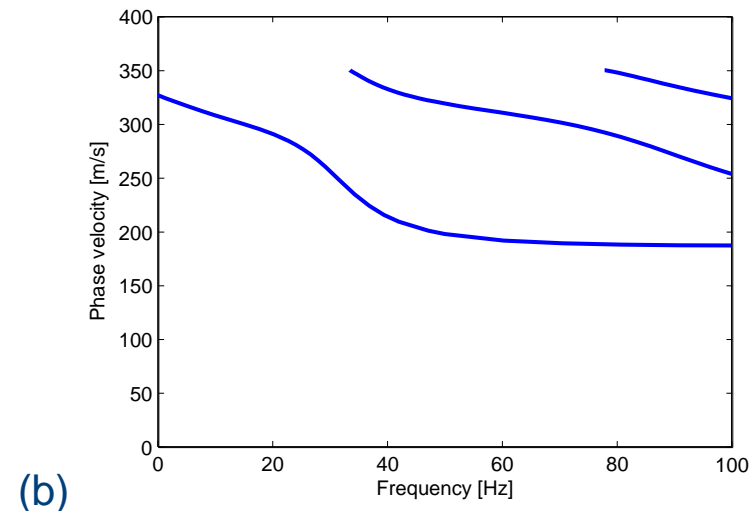
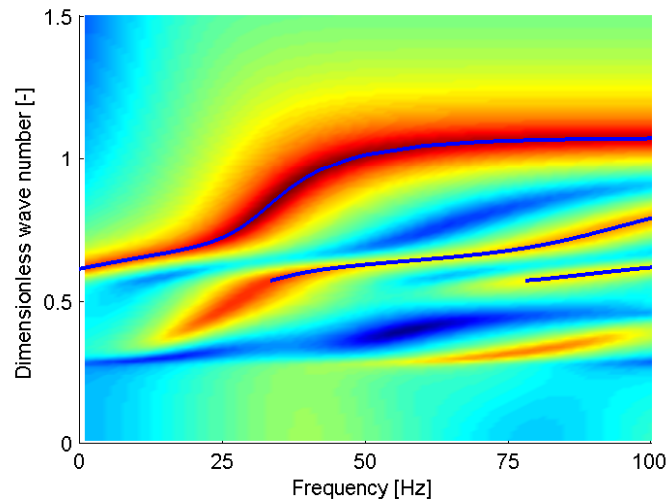
- Horizontal component, vertical component and norm of the Green's function of the layered halfspace at (a) 20 Hz, (b) 40 Hz, and (c) 80 Hz.

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## Example 2: layer on a halfspace

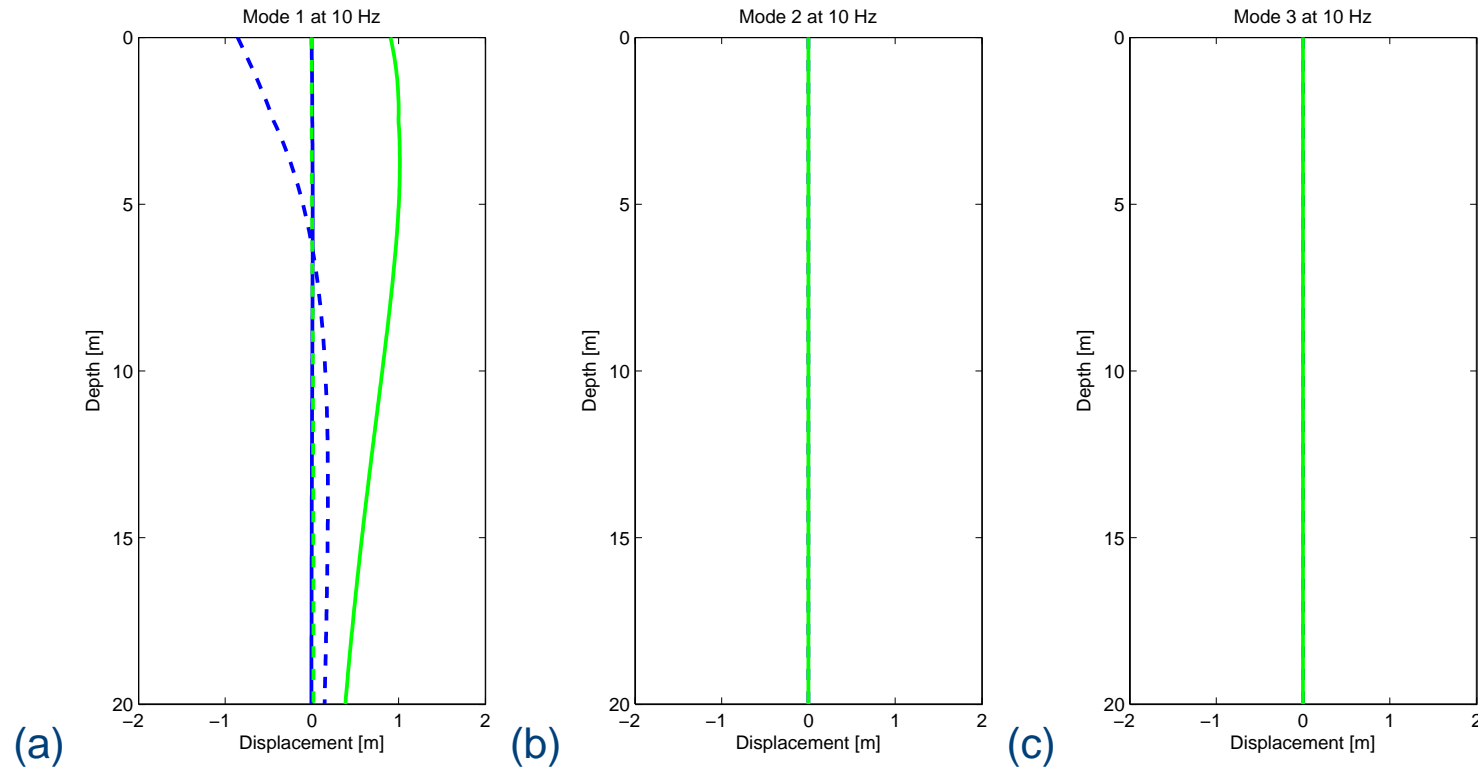
- Layer with a thickness of 2.5 m and a shear wave velocity  $C_s = 200.0$  m/s, a dilatational wave velocity  $C_p = 400.0$  m/s, a density  $\rho = 1745.0$  kg/m<sup>3</sup> and a material damping ratio  $\beta = \beta_s = \beta_p = 0.025$  on a halfspace with a shear wave velocity  $C_s = 350.0$  m/s, a dilatational wave velocity  $C_p = 700.0$  m/s, a density  $\rho = 1745.0$  kg/m<sup>3</sup> and a material damping ratio  $\beta = \beta_s = \beta_p = 0.025$
- (a) Green's function  $\log |\omega \tilde{u}_{zz}^G(\bar{k}_r, z = 0, \omega)|$  as a function of  $\omega$  and  $\bar{k}_r = k_r C_s / \omega = C_s / C_r$  and (b) dispersion curves.



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## Example 2: layer on a halfspace

- Real (solid line) and imaginary (dashed line) part of the horizontal and vertical displacement at 10 Hz in (a) mode 1, (b) mode 2, and (c) mode 3.

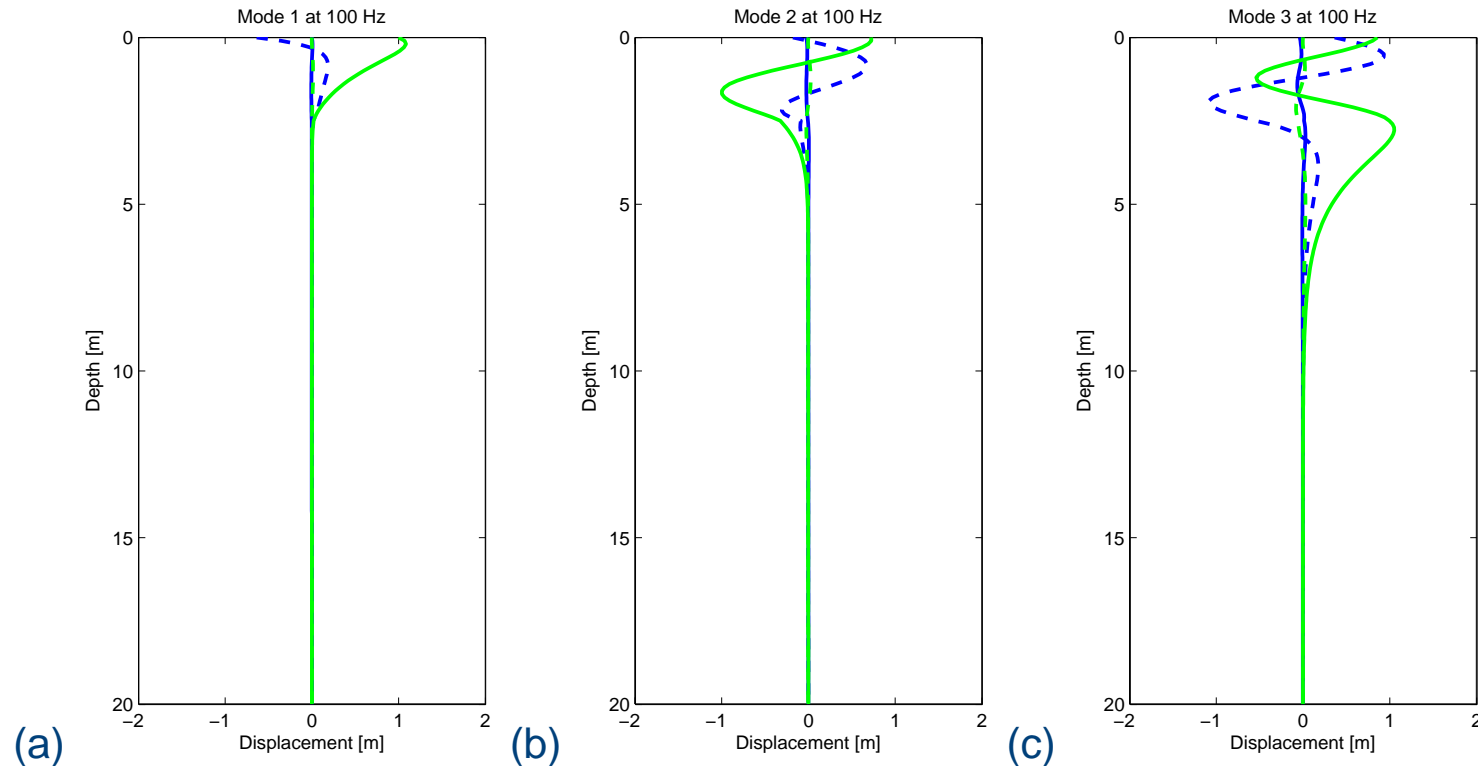


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## Example 2: layer on a halfspace

- Real (solid line) and imaginary (dashed line) part of the horizontal and vertical displacement at 100 Hz in (a) mode 1, (b) mode 2, and (c) mode 3.

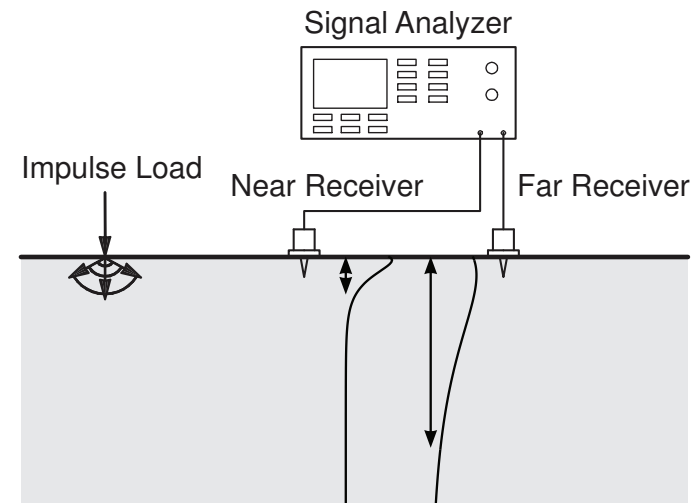


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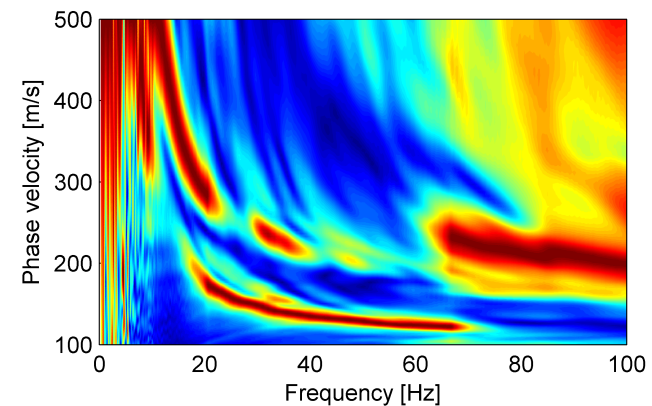
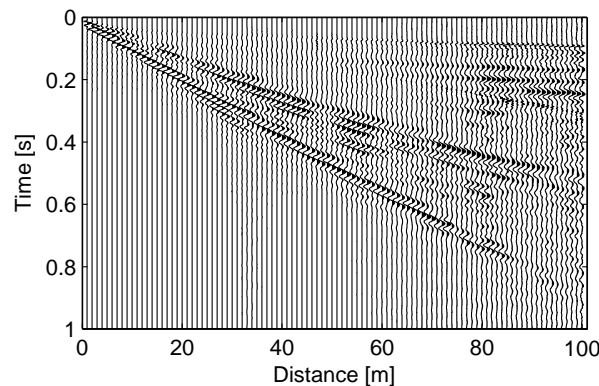
## Spectral Analysis of Surface Waves

- In situ experiment to determine the dispersion curve:

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- Response in the (a) space-time domain and (b) the frequency-wavenumber domain.



## Spectral Analysis of Surface Waves

Problem outline

Elastic wave propagation

Dynamic soil characteristics

Case history



- Complex shear modulus  $\mu^*$  (correspondance principle):

$$\mu^* = \mu(1 + 2\beta_s i) \quad (4)$$

with  $\mu$  the small strain shear modulus and  $\beta_s$  the material damping ratio.

- Transfer function in the soil (as measured):

$$\hat{H}_{zz}^E(r, \omega) = \frac{\hat{u}_z^E(r, \omega)}{\hat{F}_z^E(\omega)} \quad (5)$$

- Transfer function in the soil (mathematical representation):

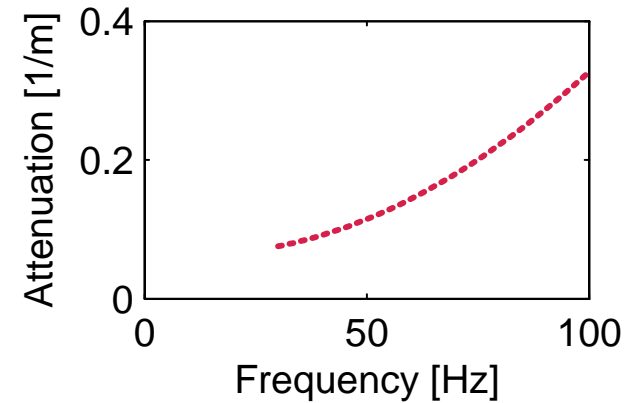
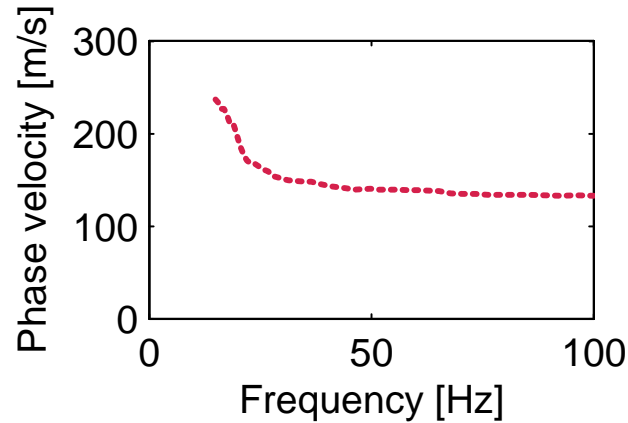
$$\hat{h}_{zz}^E(r, \omega) = \zeta(r, \omega) \exp\left(-i \frac{\omega}{C_R^E(\omega)} r\right) \exp\left(-A_R^E(\omega) r\right) \quad (6)$$

with  $\zeta(r, \omega)$  the geometric spreading factor,  $C_R^E(\omega)$  the phase velocity, and  $A_R^E(\omega)$  the attenuation coefficient.

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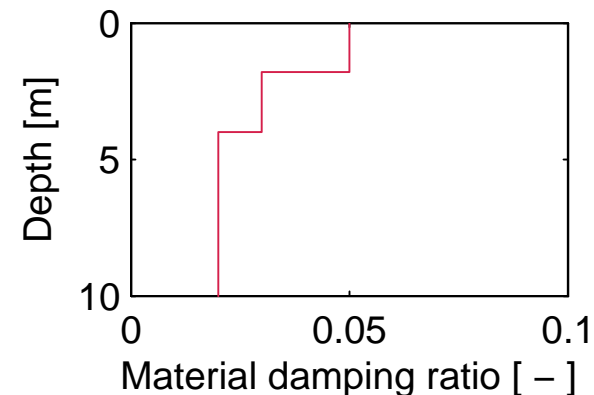
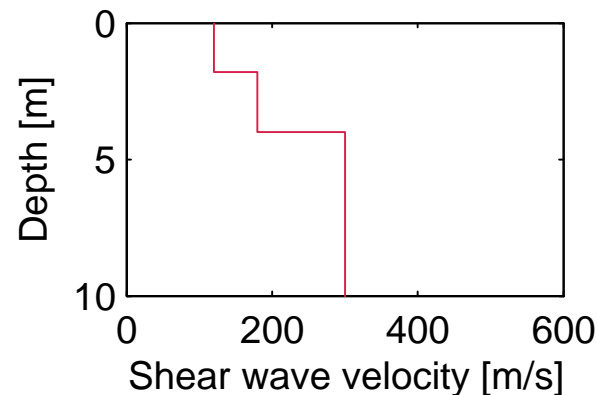
## Spectral Analysis of Surface Waves

- The **dispersion curve** and the **attenuation curve** are derived from the experiment.



- An **inverse problem** is solved to determine the dynamic soil properties corresponding to the experimental data. The soil is assumed to be horizontally layered.

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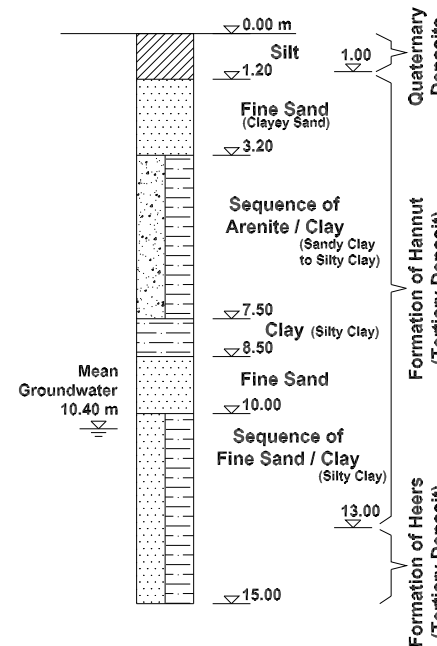
## Test site in Lincent, Belgium (HST line L2 Brussels-Köln)

Problem outline

Elastic wave propagation

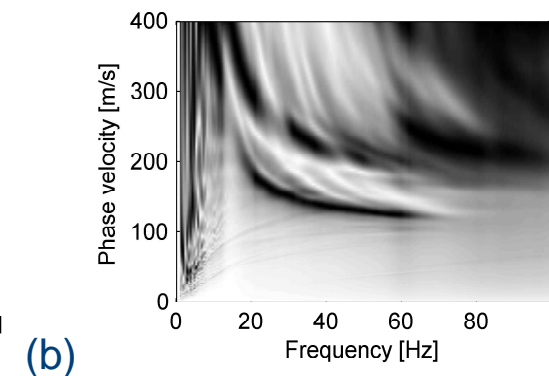
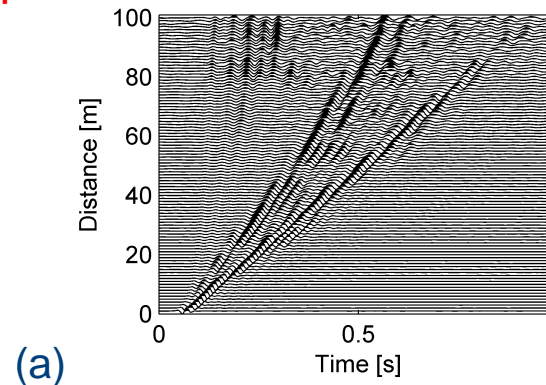
Dynamic soil characteristics

Case history



- 104 accelerometers are used from 1 m to 104 m.
- Response due to 85 hammer impacts is averaged to remove background noise.
- Fundamental dispersion curve between 15 and 70 Hz is clearly visible.

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## Seismic Cone Penetration Test

Problem outline

Elastic wave propagation

Dynamic soil characteristics

Case history

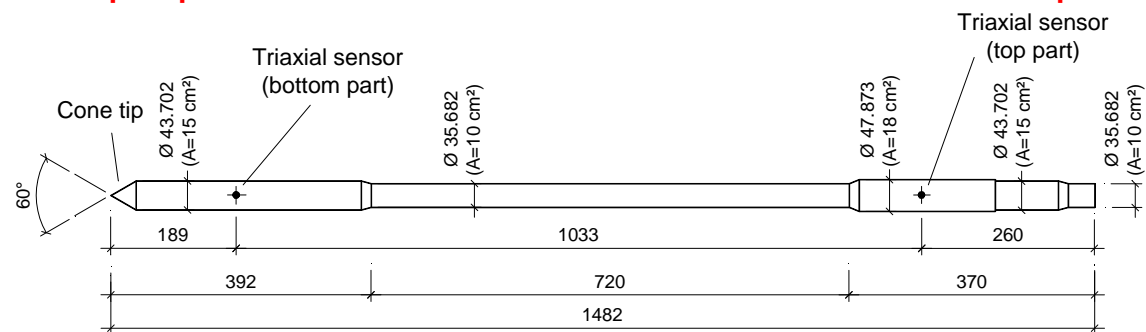


(a)



(b)

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(c)

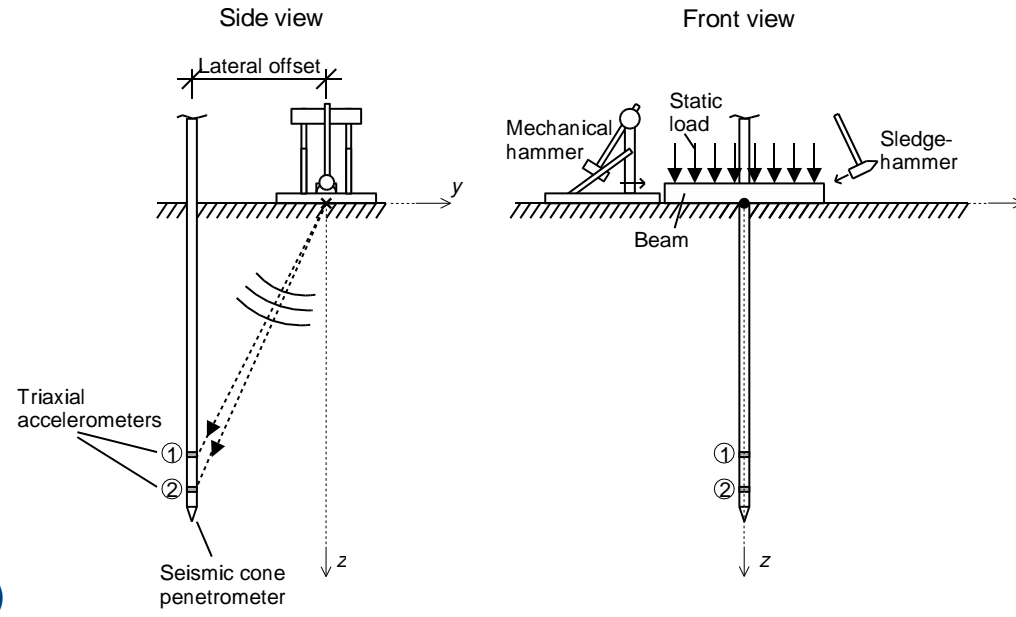
## Seismic Cone Penetration Test

Problem outline

Elastic wave propagation

Dynamic soil characteristics

Case history

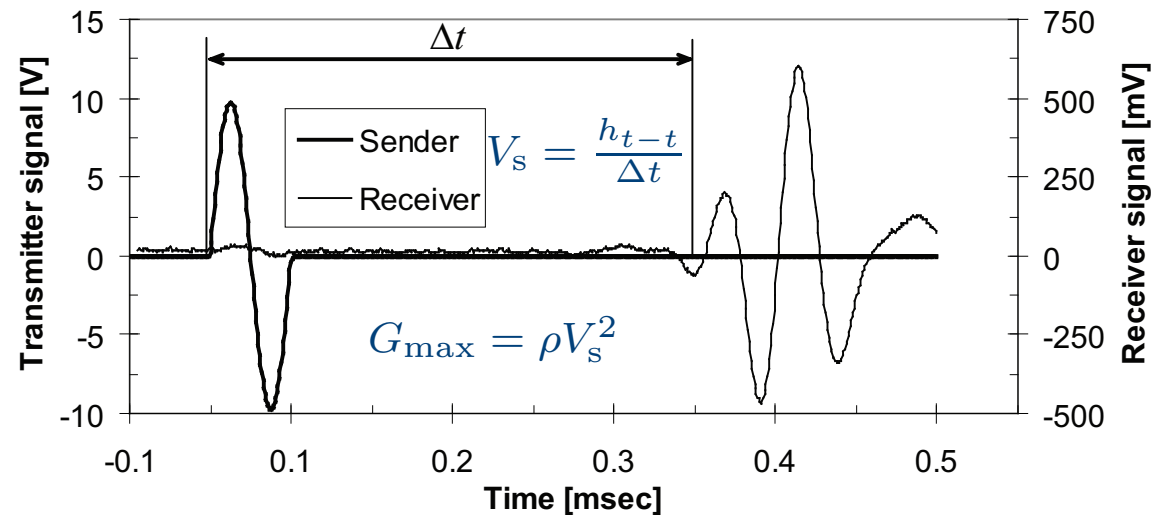
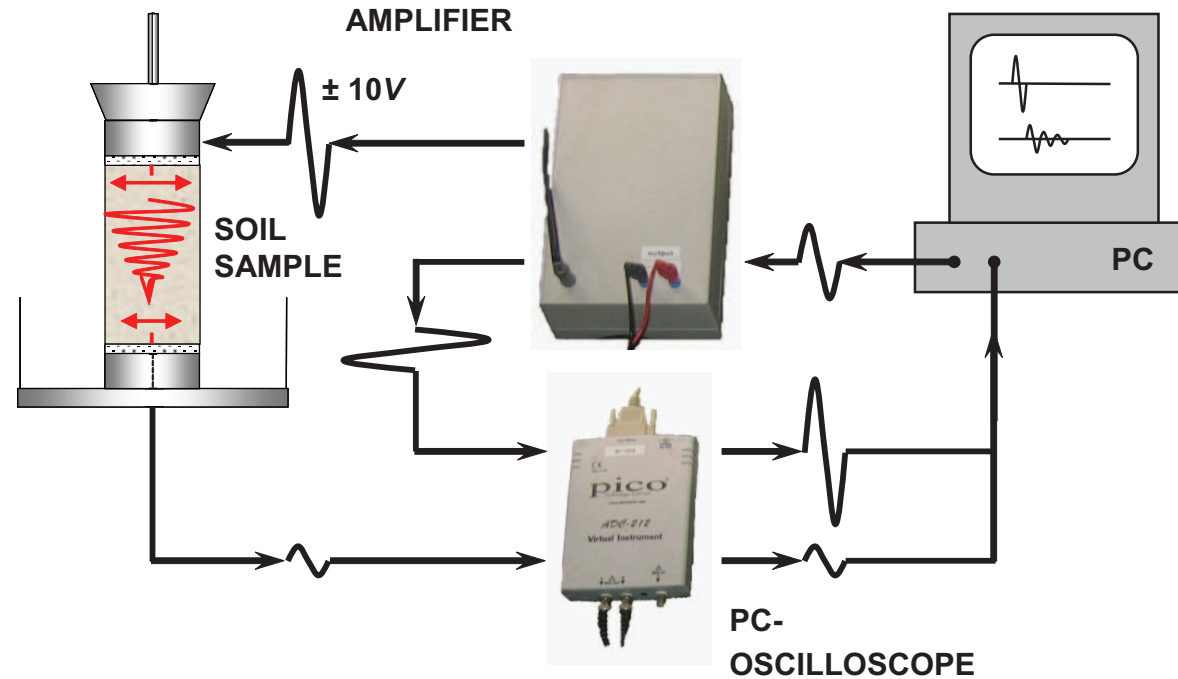


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**Bender element test**

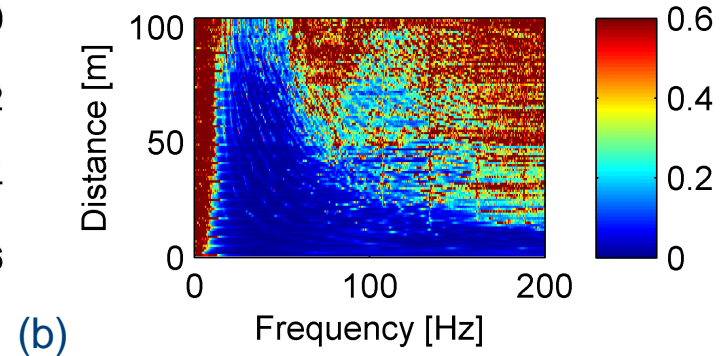
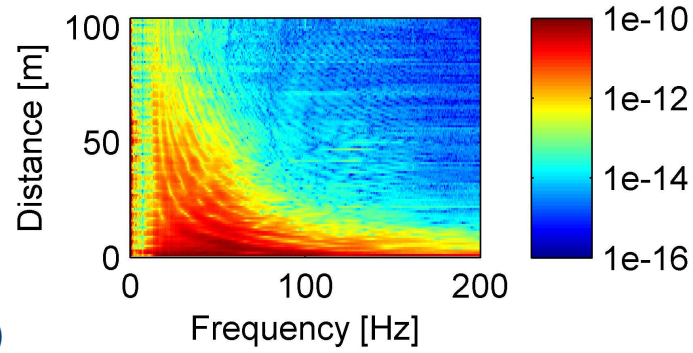
- Problem outline
- Elastic wave propagation
- Dynamic soil characteristics
- Case history



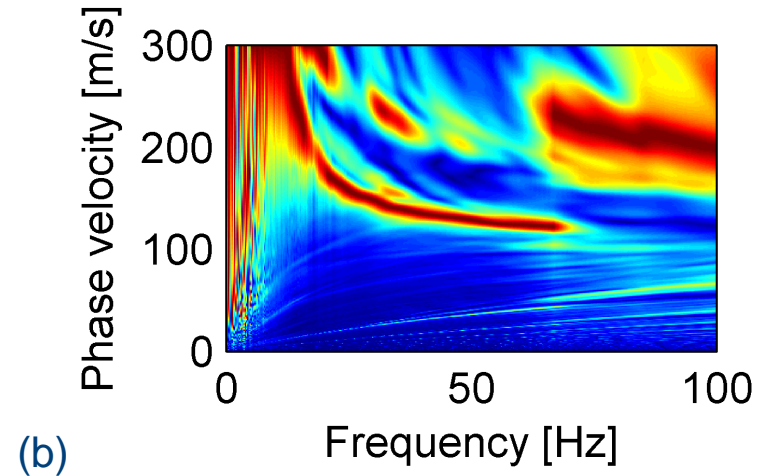
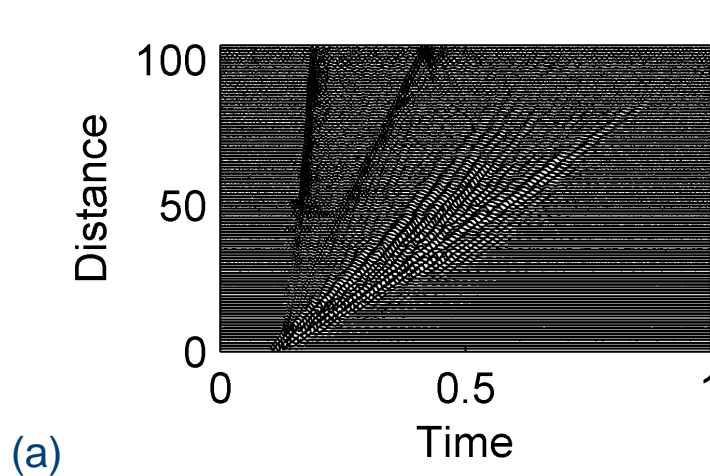


## Dynamic soil characteristics

- (a) Transfer function  $\hat{H}(r, \omega)$  and (b) coefficient of variation  $\hat{\sigma}_H^2(r, \omega) / \hat{H}(r, \omega)$ .

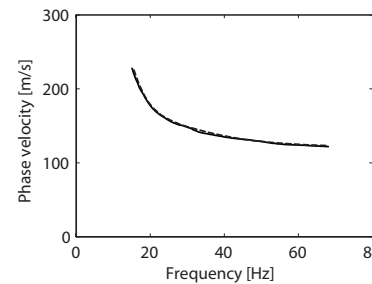


- Response in the (a) time-space and (b) frequency-wavenumber domain.

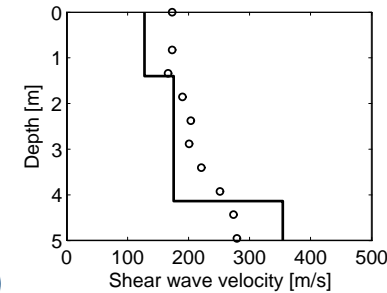


## Dynamic soil characteristics

- (a) Dispersion curve and (b) shear wave velocity profile.

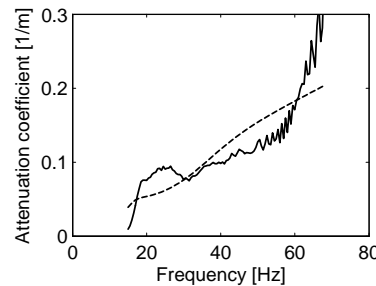


(a)

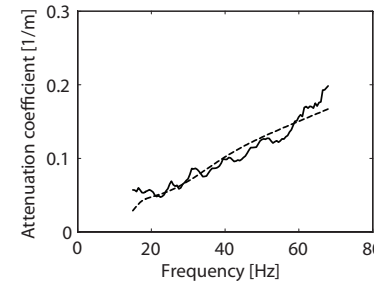


(b)

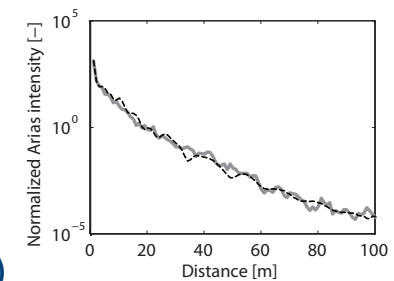
- (a,b) Attenuation curve (methods 1 and 2) and (c) Arias intensity (method 3).



(a)

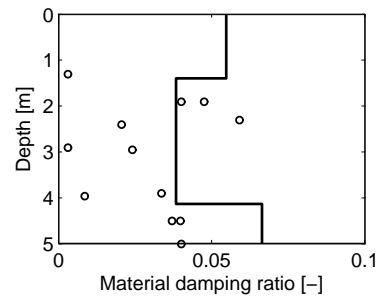


(b)

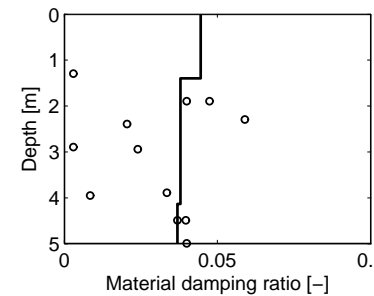


(c)

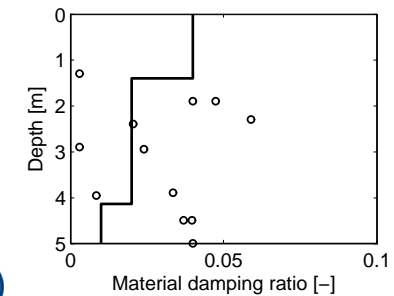
- Material damping ratio profile determined with (a) method 1, (b) method 2, and (c) method 3.



(a)



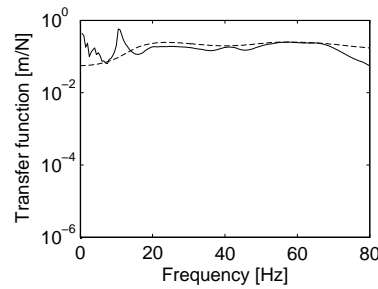
(b)



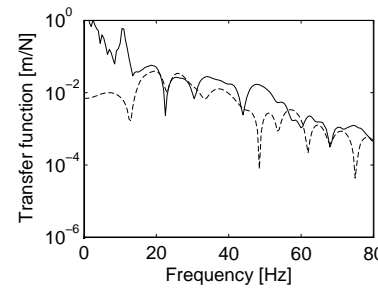
(c)

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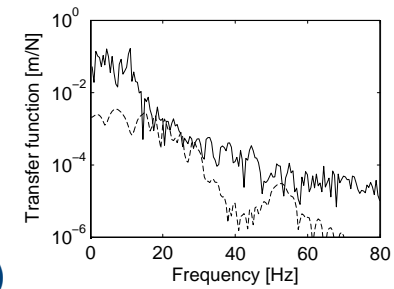
- Measured (solid line) and computed (dashed line) transfer function at (a) 5 m, (b) 30 m, and (c) 100 m. The material damping ratio profile is determined using:
  - ◆ method 1 (frequency-wavenumber analysis and amplitude regression):



(a)

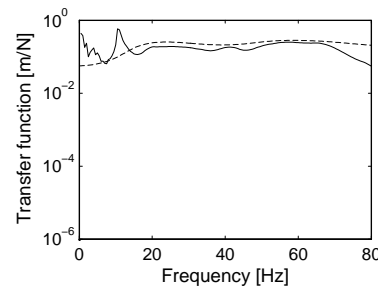


(b)

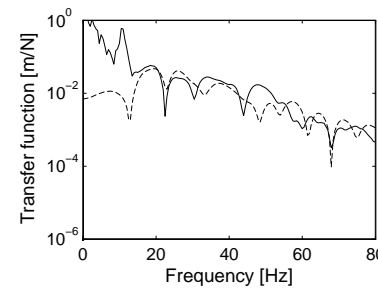


(c)

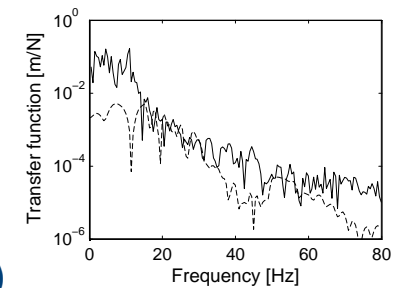
- ◆ method 2 (half-power bandwidth method):



(a)

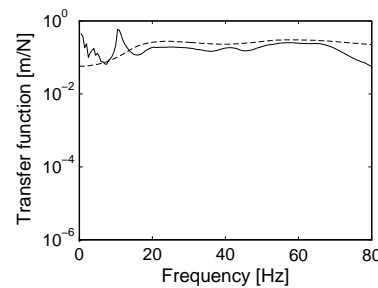


(b)

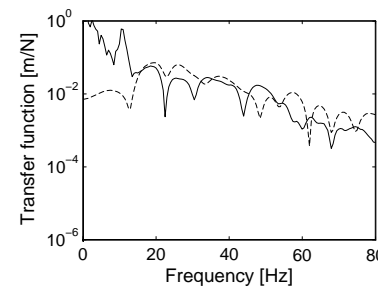


(c)

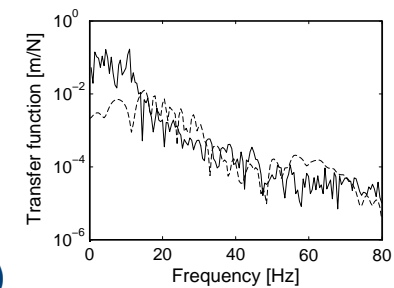
- ◆ method 3 (Arias intensity):
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(a)



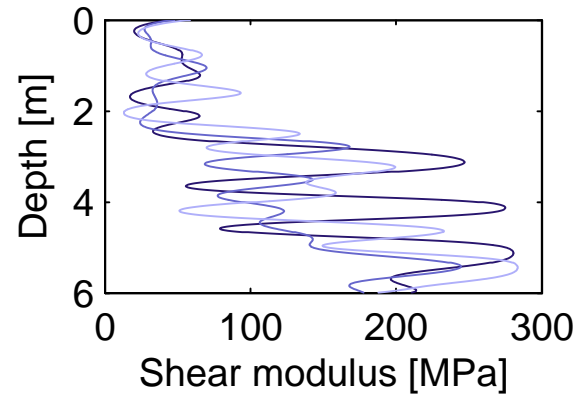
(b)



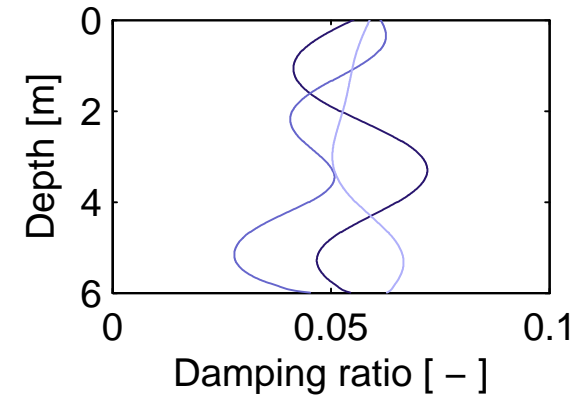
(c)

## Stochastic approach

- Three realizations of the (a) shear wave velocity and (b) damping ratio profile.



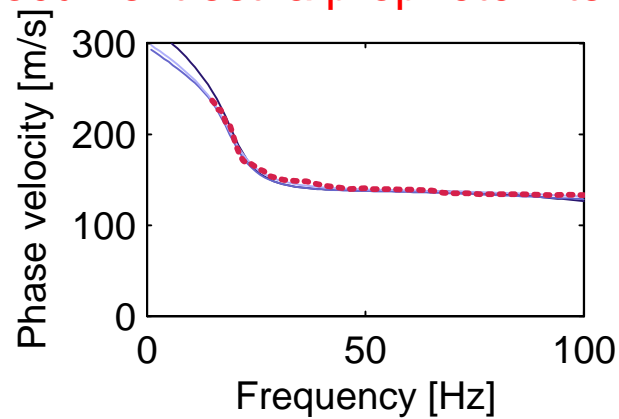
(a)



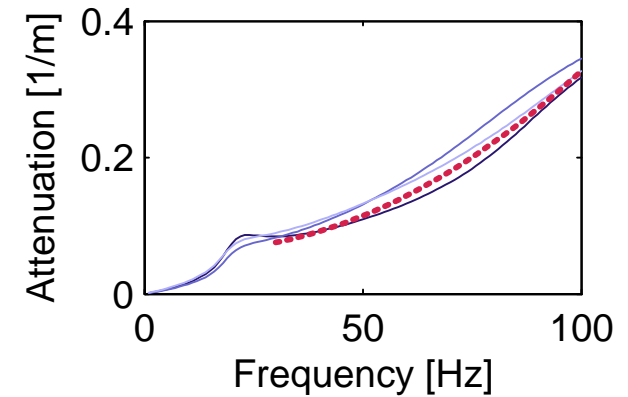
(b)

- Corresponding (a) dispersion and (b) attenuation curves.

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(a)

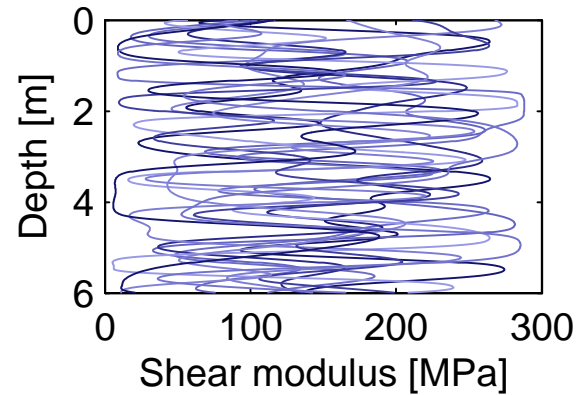


(b)

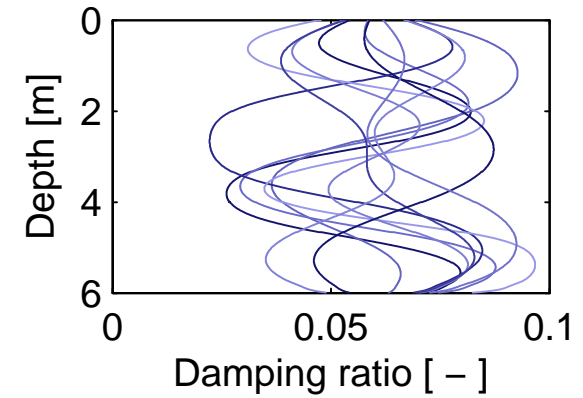
- The solution of the inverse problem is not unique  $\Rightarrow$  uncertain soil profile.
- This uncertainty affects the prediction of ground vibrations  $\Rightarrow$  robustness ?

## Stochastic approach

- Ten realizations of the prior (a) shear wave velocity and (b) material damping ratio profile.



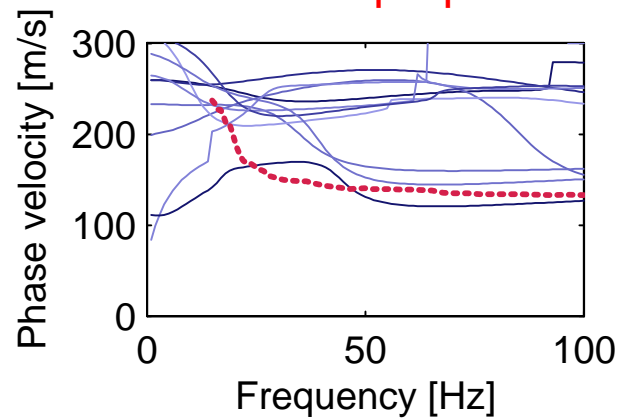
(a)



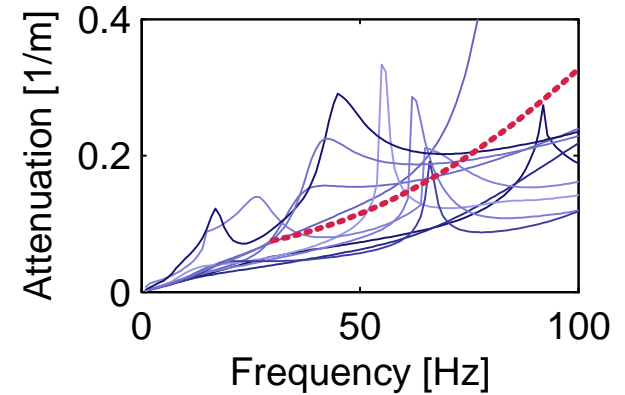
(b)

- Corresponding (a) dispersion and (b) attenuation curves.

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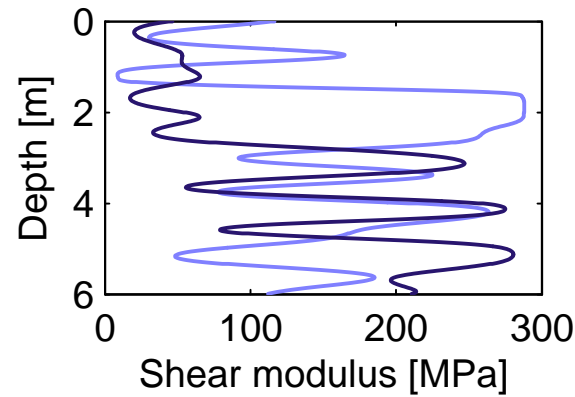
(a)



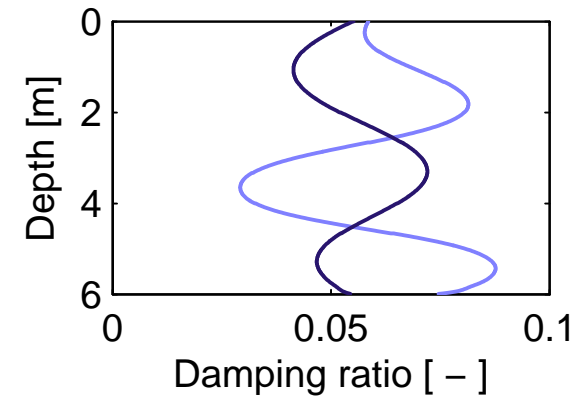
(b)

## Stochastic approach

- An acceptable and an unacceptable (a) shear wave velocity and (b) material damping ratio profile.



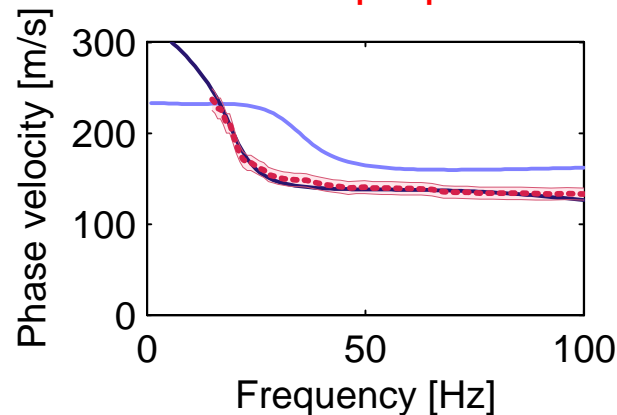
(a)



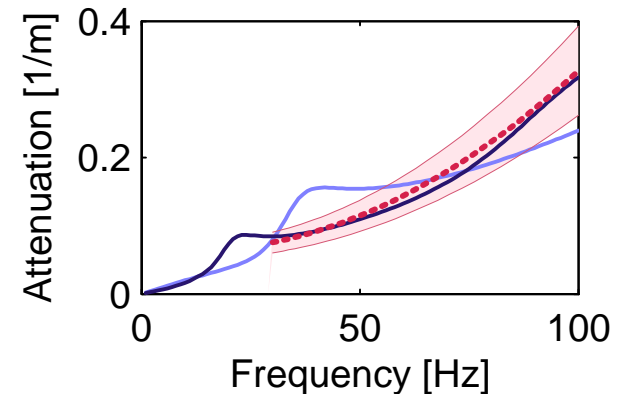
(b)

- Corresponding (a) dispersion and (b) attenuation curves.

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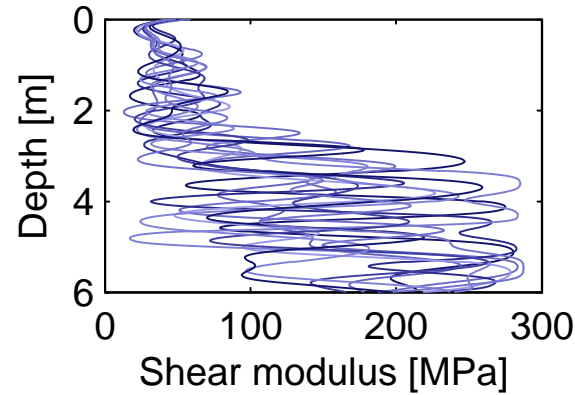
(a)



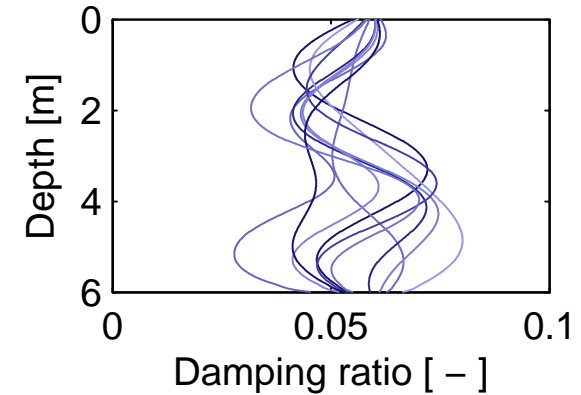
(b)

## Stochastic approach

- Ten realizations of the posterior (a) shear wave velocity and (b) material damping ratio profile.



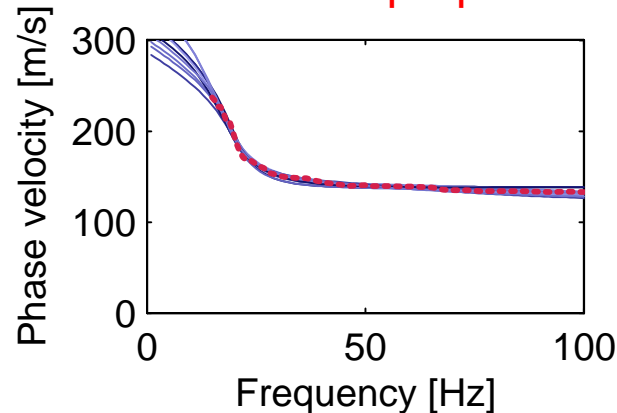
(a)



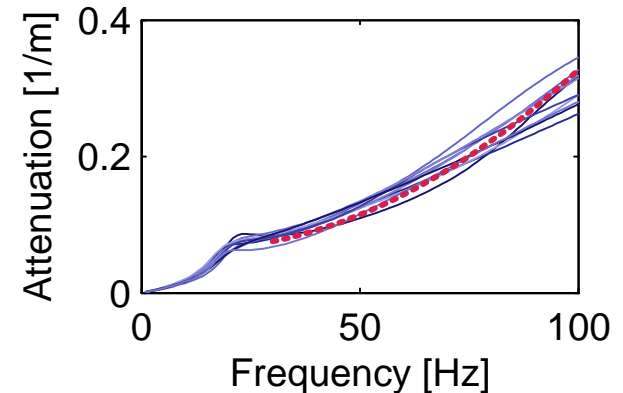
(b)

- Corresponding (a) dispersion and (b) attenuation curves.

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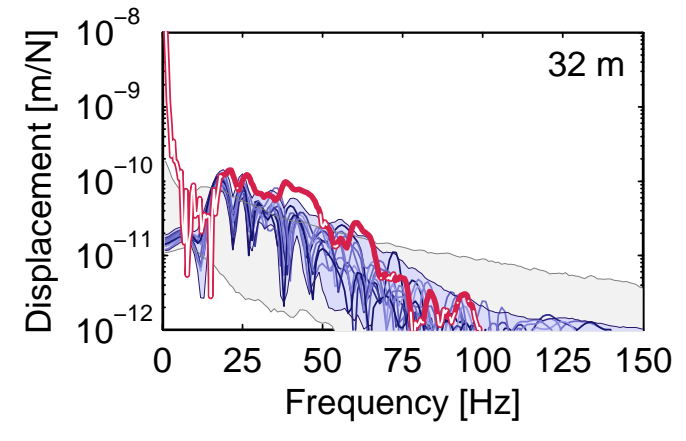
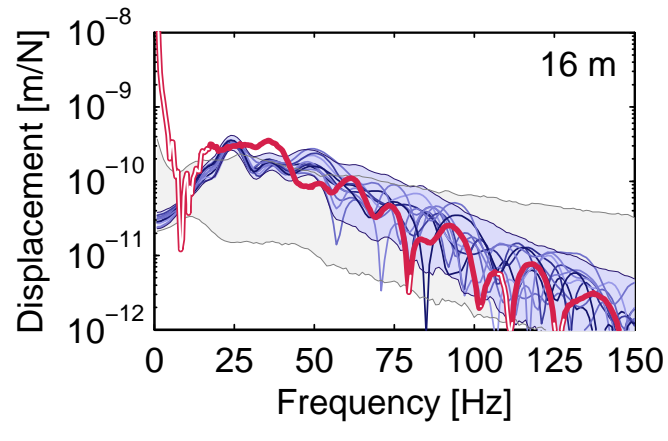
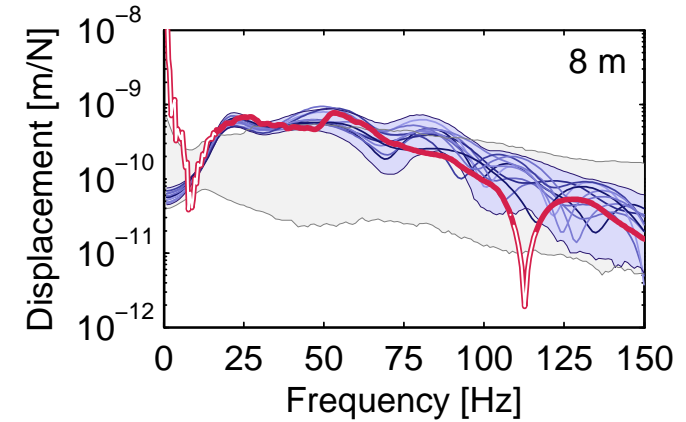
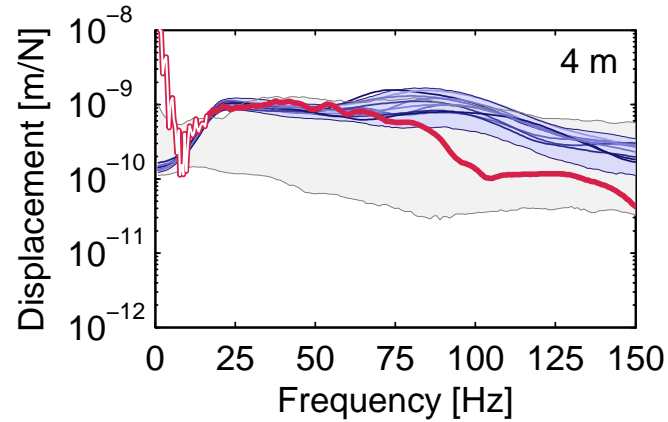
(a)



(b)

## Stochastic approach

- Predicted and **measured** response at (a) 4 m, (b) 8 m, (c) 16 m, and (d) 32 m from the foundation.



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